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LETTER TO THE EDITOR

Non-universality in the dynamics at the percolation threshold

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Abstract. The dynamical behaviour of the two-dimensional Ising model at the site percolation threshold on $L \times L$ lattices with $L \leqslant 64$ is studied by Monte Carlo simulations. The results are consistent with the recently proposed singular dynamic scaling. The new dynamical exponents are estimated to be $A_s = 0.62 \pm 0.12$ and $B_s = 3.92 \pm 0.89$. Comparison with bond dilution strongly suggests that universality is violated for this model.

Recently there has been considerable interest in the critical dynamics of diluted Ising systems at, or near, the percolation threshold for dimensions greater than one [1-6]. Both analytic [2, 3] and computational [4-6] work implies that conventional dynamic scaling [7] breaks down at low temperatures: the logarithm of the relaxation time depends quadratically upon the logarithm of the thermal correlation length. Although most of the work has been performed on various two-dimensional models, recently [6] a site-diluted three-dimensional Ising system has also been studied.

Universality, which is a fundamental concept in phase transitions, implies that the critical behaviour depends only on the dimensionality and the symmetry of the problem, i.e. it is not influenced by the fine detail of the model such as site or bond percolation. Indeed, for the two-dimensional Ising model at the percolation threshold the static critical exponents, which are believed to be known exactly [8], are the same for site and bond percolation [9].

In this letter we present the data from Monte Carlo simulations of the two-dimensional Ising model on a square lattice at the site percolation threshold. On comparing the results for the dynamics with those obtained earlier for the same model at the bond percolation threshold [5], we shall find evidence that one of the new dynamical exponents is non-universal.

The Hamiltonian is given by [9]

$$H = -\sum_{\langle ij \rangle} \eta_i \eta_j S_i S_j \tag{1}$$

where $S_i = \pm 1$, the sum runs over nearest neighbours only and the η_i are quenched site disorder variables with probability distribution

$$P(\eta_i) = (1-p)\delta(\eta_i) + p\delta(\eta_i - 1) \tag{2}$$

where p is the site concentration.

Throughout this letter the temperature is given in units of the nearest-neighbour exchange interaction, which is set equal to unity (as is Boltzmann's constant). The simulations were performed on $L \times L$ ($L = 16, 32$ and 64) lattices with periodic boundary conditions in both directions and the site percolation threshold was taken to be $p_c = 0.5927$ [10].

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The computational work was carried out on the distributed array processor (DAP) at Queen Mary College, London. The speed of the program depends, among other factors, upon L and the number of Monte Carlo steps per spin (MCS/spin) performed; the fastest speed we were able to achieve was approximately 5.8 million updates per second.

As the temperature, T, is lowered, we find that both the thermal correlation length (see later) and relaxation times increase much more rapidly than for the bond percolation model [5]. As a consequence, the temperature range studied was restricted to $0.8 \leqslant T \leqslant 2.0$ ($0.35 \leqslant T/T_c \leqslant 0.88$, where [11] $T_c(P=1) \simeq 2.27$ in our units). At $T=0.8$ the system only achieved equilibrium after the first 10^6 MCS/spin. For $T \geqslant 0.9$ the data presented have been averaged over 75-238 samples and the statistical error bars are typically $\leqslant 10\%$; for $T=0.8$, however, only fourteen samples were used and the error is about $22\frac{1}{2}\%$.

As the spins are all pointing up at the start of the simulation, the magnetisation $M(t)$, given by

$$M(t) = N^{-1} \sum_i S_i(t) \tag{3}$$

where $S_i(t)$ is the value of the ith spin at time t and N is the number of spins, is identically equal to one. The system is then allowed to evolve according to Glauber dynamics [12]. The decay of $M(t)$ is not exponential and at low T it is close to logarithmic [6]. After an equilibration time, $t=\tau_0$, the system attains equilibrium [5, 9]: $M(t=\tau_0)=0$. In order to investigate the dynamics of the model, we first look at the autocorrelation function

$$C(t) = N^{-1} \sum_i S_i(t_0) S_i(t+t_0) \tag{4}$$

where t_0 is an initial time and $t > t_0$. $C(t)$ is seen to decrease with t and $C(t)=0$ for $t > t_0$. For each temperature one chooses $t_0 \geqslant \tau_0$. In figures 1(a) and (b) we examine the decay of $C(t)$. Clearly, the plot shown in figure 1(b) gives the better fit, i.e. the autocorrelation function decays algebraically over a broad time interval. An average relaxation time, τ_{AV}, is defined by [13]

$$\tau_{AV} = \int_0^\infty C(t)\, dt. \tag{5}$$

At $p=p_c$ one has an infinite cluster spanning the whole lattice as well as finite clusters [9]. One would not expect the various clusters to relax at the same rate. τ_{AV} averages over all the different relaxation rates and is a measure of the largest relaxation time of the system. Information concerning the decay of correlations with distance is obtained from

$$\Gamma(n) = N^{-1} \sum_i \langle S_i S_{i+n} \rangle_T \tag{6}$$

where $\langle \ldots \rangle_T$ denotes a thermal average and $n (n = 0, 1, \ldots, 16)$ is the displacement in the x direction. Statistically reliable data for $\Gamma(n)$ were obtained for $1.0 \leqslant T \leqslant 2.0$ ($0.44 \leqslant T/T_c \leqslant 0.88$). In figure 2 we plot $\ln \Gamma(n)$ against n for various temperatures in the interval $1.0 \leqslant T \leqslant 2.0$ with $L=64$. Hence we have

$$\Gamma(n) \sim \exp(-n/\xi_T) \tag{7}$$

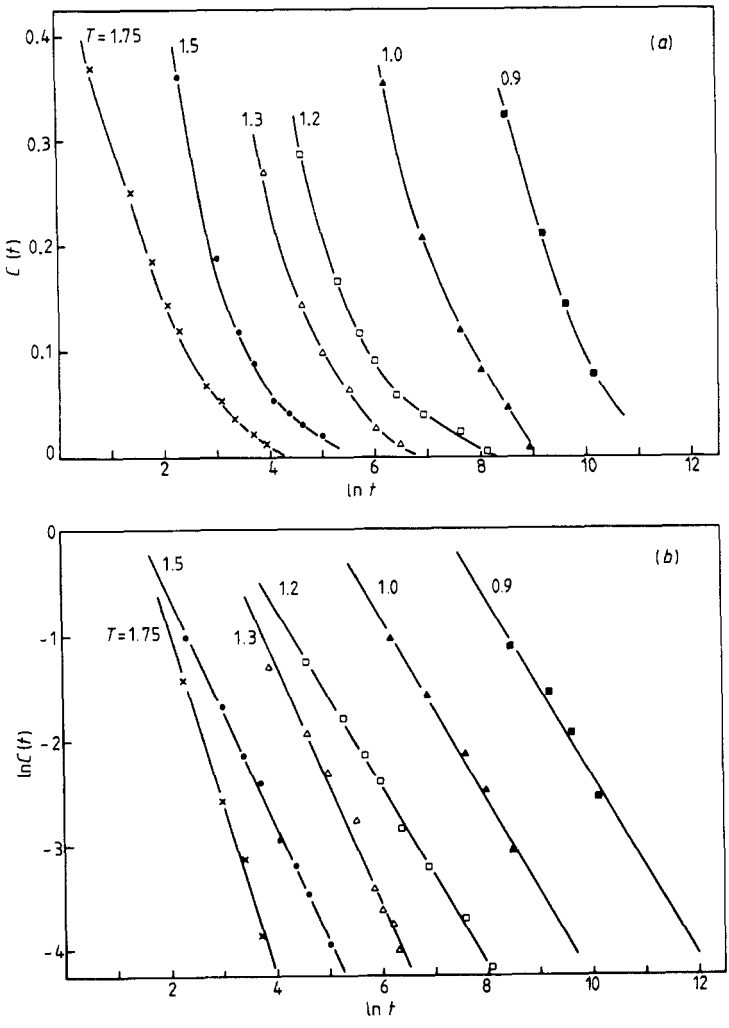

Figure 1. (*a*) Relaxation of the autocorrelation function with ln t ($L = 64$). The full curves are guides for the eye. (*b*) ln $C(t)$ plotted against ln t. The data for $T = 1.75$ are for $L = 32$, whereas the rest are for $L = 64$. The good linear fits indicate that $C(t)$ decays algebraically. The curves (gradient $\simeq -1$) are guides for the eye.

for $n \gg \xi_T$, where ξ_T is the thermal correlation length, allowing the latter to be extracted. ξ_T is expected to behave as

$$\xi_T = \xi_s \exp(2\nu_T/T) \tag{8}$$

where ν_T is the thermal exponent and ξ_s is the amplitude. In figure 3 we plot ln ξ_T against $1/T$ for $1.0 \leqslant T \leqslant 2.0$ (here $L = 64$). As one expects equation (8) to be valid in the scaling regime where $\xi_T \geqslant 1$ (the lattice spacing), the weighted line of best fit is also shown; from the gradient and intercept one obtains $\nu_T = 1.31 \pm 0.04$ and $\xi_s = 0.31 \pm 0.03$. The value for ν_T agrees well (within the error bars) both with the result obtained recently for the bond percolation problem [5] and the conjectured exact result [8] of $\frac{4}{3}$. The amplitudes in the two cases, however, differ and

$$\xi_s/\xi_b = 1.42 \pm 0.20 \tag{9}$$

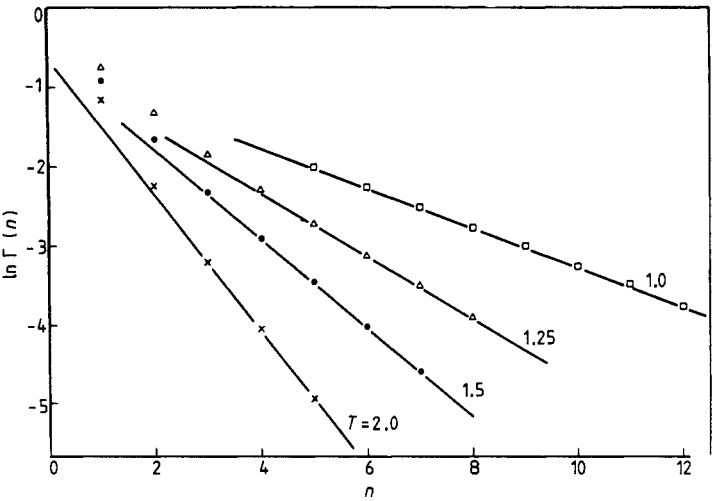

Figure 2 Logarithm of the correlation function against distance for several temperatures. The straight lines are used to extract ξ_T (see text).

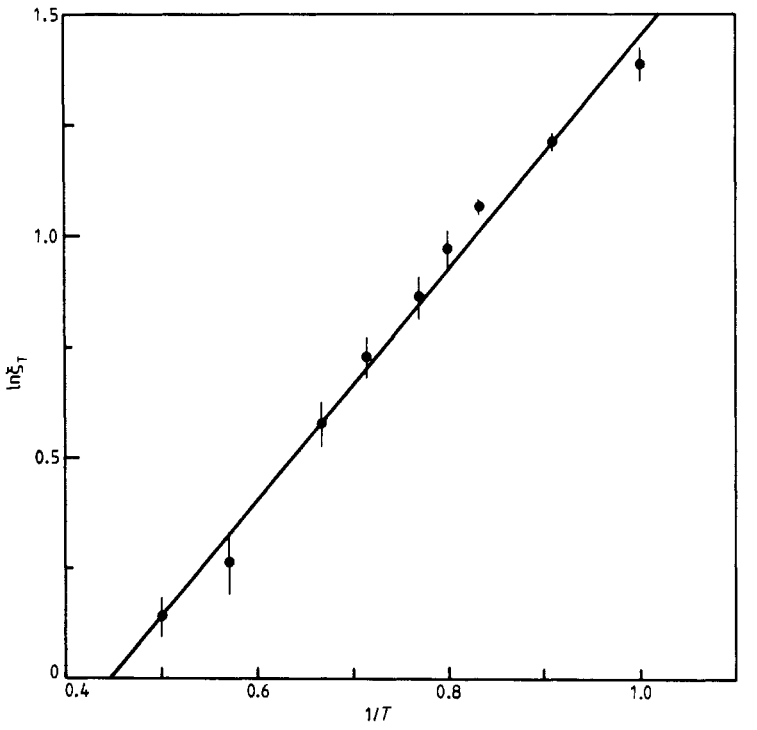

Figure 3. A plot of ln ξ_T against $1/T(1.0 \leq T \leq 2.0)$ for a 64×64 lattice at the site percolation threshold. As explained in the text, the weighted line of best fit (also shown) gives $\nu_T = 1.31 \pm 0.04$ and $\xi_s = 0.31 \pm 0.03$.

where ξ_b is the amplitude for the bond percolation model [5] (note that in the notation of [5] the amplitude is given by ξ_0). As a consequence, ξ_T increases much faster for the site percolation case. For example, ξ_T $(T=1.0)=3.99$ and by extrapolation one has ξ_T $(T=0.8)=8.69$. Despite the rapid increase in the thermal correlation length, one still has $\xi_T \ll L$ for all temperatures and sizes considered. Consequently, finite-size effects are negligible—this is confirmed explicitly for the dynamics.

In the dynamic scaling hypothesis one has

$$\ln(\tau_{AV}(T)) = f(\ln \xi_T). \tag{10}$$

Conventional dynamic scaling [7], $f(x)=Zx$ (here Z is a dynamic critical exponent), breaks down for this model and one instead has [2-6]

$$f(x) = Ax^2 + Bx + C \tag{11}$$

where A and B are new exponents and C is a temperature-independent constant which sets the microscopic timescale. From equations (8), (10) and (11) we can write

$$\ln(\tau_{AV}(T)) = \alpha_1/T^2 + \alpha_2/T + \alpha_3 \tag{12}$$

where $\alpha_1 = 4A\nu_T^2$, $\alpha_2 = 2\nu_T$ $(B+2A\ln\xi_s)$ and $\alpha_3 = C + B\ln\xi_s + A(\ln\xi_s)^2$. In figure 4 we display the results for $\ln\tau_{AV}$ against $1/T$ $(0.8 \leqslant T \leqslant 2.0)$ for the site percolation model for various values of L. As mentioned above, there are no apparent finite-size effects. Assuming equation (12) to hold, we also show the best quadratic fit to the

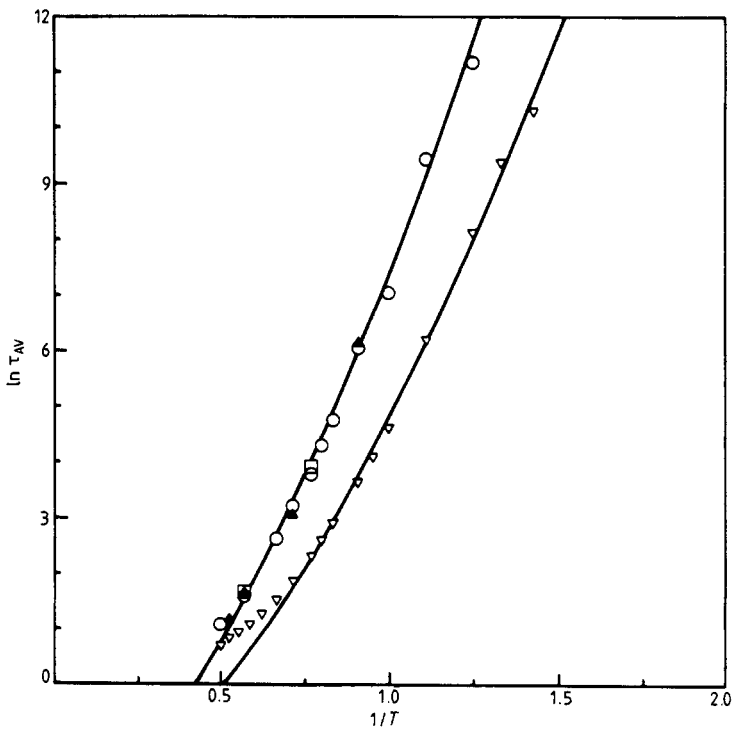

Figure 4. Here we show $\ln\tau_{AV}$ against $1/T$. The data for the site percolation model are displayed by circles (64×64), squares (32×32) and full triangles (16×16); for comparison we also show the data (open triangles) obtained previously [5] for the bond percolation problem on a 64×64 lattice. The best quadratic fit in each case is also shown.

data. For comparison, also shown are the data for the bond percolation case [5]. In both cases singular dynamic scaling (equation (12)) is seen to hold at low temperatures. For the data shown we have $A_s = 0.62 \pm 0.12$, $B_s = 3.92 \pm 0.89$ and $A_b = 0.51 \pm 0.05$, $B_b = 3.25 \pm 0.41$ [5], where the subscripts s and b refer to the site and bond percolation cases, respectively. We note that recently [3] $A_b \sim 2.16$ was obtained by applying a decimation procedure to the bond percolation model on a square lattice. As this method gives neither the correct static percolation exponent nor the known exact value of p_c ($=\frac{1}{2}$) [14], we have reservations about this large value for A_b.

We can also consider

$$\ln(\tau^s_{AV}/\tau^b_{AV}) = \beta_1/T^2 + \beta_2/T + \beta_3 \tag{13}$$

where the superscripts once again refer to site and bond percolation and $\beta_1 = 4\nu_T^2(A_s - A_b)$, $\beta_2 = 2\nu_T[(B_s - B_b) + 2(A_s \ln \xi_s - A_b \ln \xi_b)]$ and $\beta_3 = (C_s - C_b) + (B_s \ln \xi_s - B_b \ln \xi_b) + [A_s(\ln \xi_s)^2 - A_b(\ln \xi_b)^2]$. Hence, if the exponent A is universal ($A_s = A_b$) one would expect a plot of $\ln(\tau^s_{AV}/\tau^b_{AV})$ against $1/T$ to be linear. Further, if B is also universal ($B_s = B_b$) one would expect the slope of the linear fit to be

$$\beta_2(A_s = A_b = A,\ B_s = B_b) = 4\nu_T A \ln(\xi_s/\xi_b)$$
$$= 1.23 \pm 0.86 \tag{14}$$

where the numerical value has been obtained by assuming equation (9) and that $0.46 \leqslant A \leqslant 0.74$. Figure 5 shows a plot of $\ln(\tau^s_{AV}/\tau^b_{AV})$ against $1/T$. We obtain a

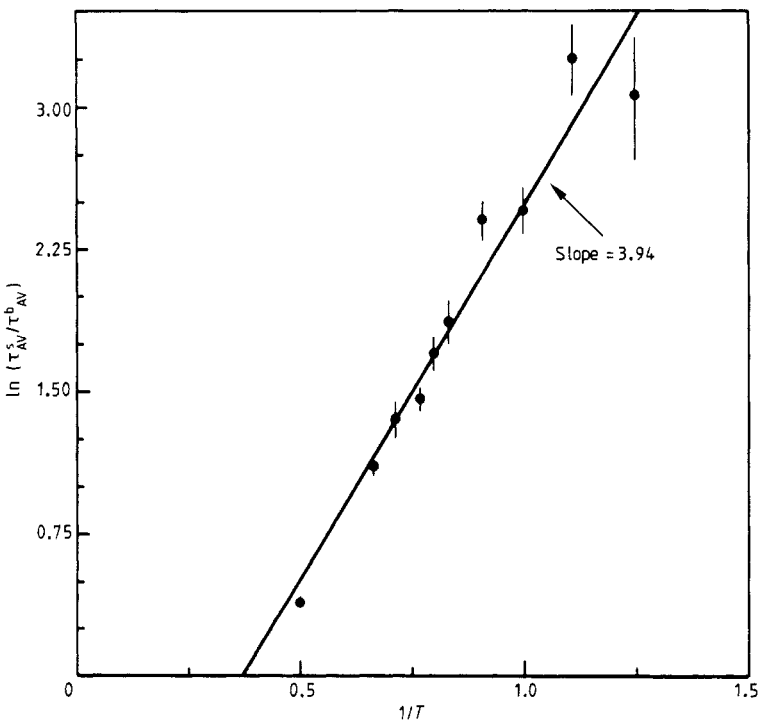

Figure 5. The data shown in figure 2 are replotted here as $\ln(\tau^s_{AV}/\tau^b_{AV})$ against $1/T$. The best linear fit has slope 3.94 ± 0.37 and intercept -1.47 ± 0.2; if B were universal one would expect a slope of 1.23 ± 0.86.

reasonable linear fit over a broad temperature range. So this would tend to confirm that A is universal. Further, the line of best fit (which is also shown in figure 5) has gradient $= 3.94 \pm 0.37$, which is inconsistent with equation (14). As a consequence, we suggest that if A is universal (which seems likely) then B is non-universal between bond and site percolation.

To conclude, we have shown that the two-dimensional Ising model at the site percolation threshold has singular dynamic scaling. The autocorrelation function has been shown to decay algebraically with time. Further, on comparing our results with those obtained for the bond percolation problem, we find strong evidence which supports the view that, while A is possibly universal, the dynamic exponent B is non-universal.

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